On
TAEs in Tokamak Plasmas
and
Solar Flares-CMEs in Solar Corona

2017 S. Chandrasekhar Prize Lecture

by

C. Z. (Frank) Cheng[1,2,3]

[1] **Physics Department, Lehigh University, USA**

[2] **National Cheng Kung University**

[3] **University of Tokyo**

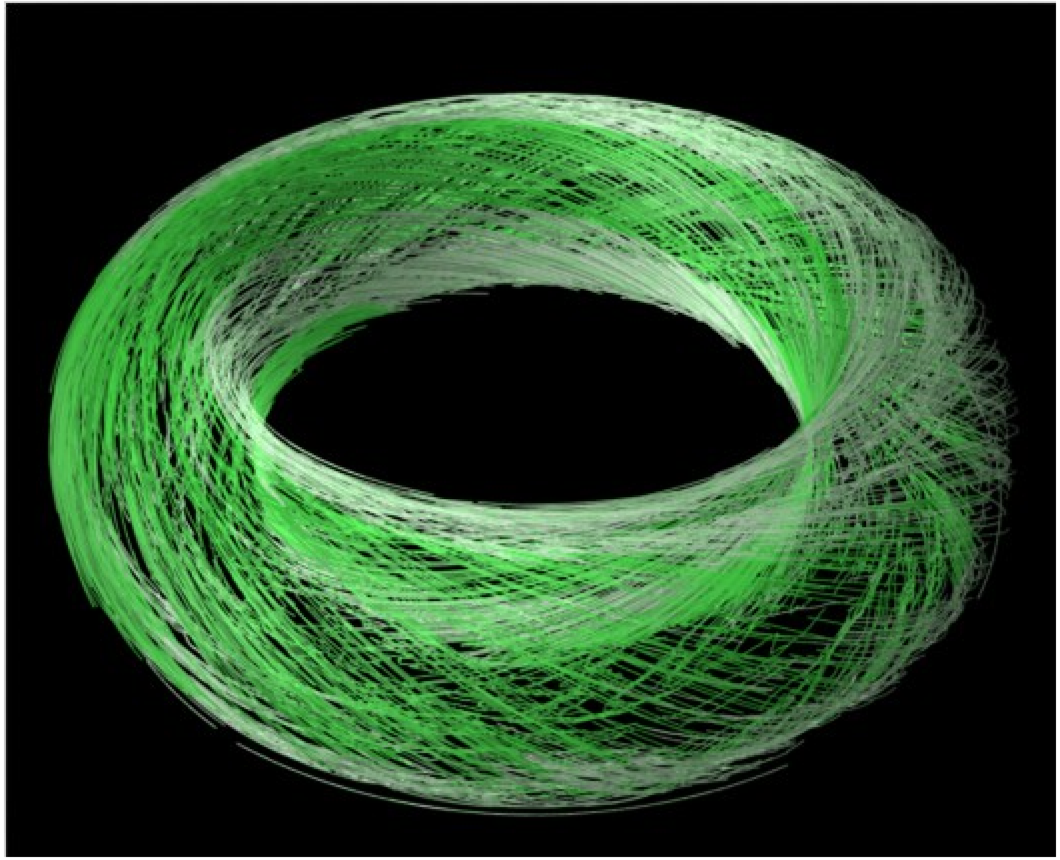
Citation of
S. Chandrasekhar Prize in Plasma Physics:

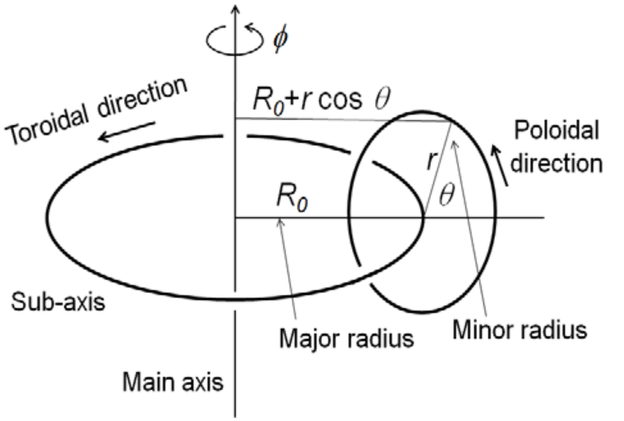
For original and pioneering contributions in fusion and space plasmas that include the theoretical discovery of Torodicity-Induced Alfven Eigenmodes, the invention of a splitting scheme for Vlasov simulation, pioneering three-dimensional Particle-In-Cell turbulence simulations in tokamaks, and the establishment of magnetized plasma experimental capability and space instrumentation development in Taiwan.

Outline

- **TAEs & fast ion/alpha confinement**
  - **MHD continuous spectrum & gaps in tokamaks**
  - **Existence of TAEs in continuum gap**
  - **TAE instabilities and loss of fast ions/alphas**
  - **Burning plasma physics in fusion reactor**
- **Magnetic reconnection in solar and laboratory plasmas**
  - **Magnetic reconnection in solar flares-CMEs**
  - **CME(plasmoid) Acceleration causes enhanced reconnection rate**
  - **Reconnection in merging plasma experiments**
  - **Plasma heating/acceleration by magnetic reconnection in collisionless plasmas**

Tokamak Magnetic Field





Tokamak magnetic field: $\vec{B} = \vec{B}_t + \vec{B}_p$

Field line equation: $\dfrac{r d\theta}{B_p} = \dfrac{R d\varphi}{B_t}$

Force Equilibrium: $\vec{J} \times \vec{B} = \nabla P$

Field lines form closed surfaces.

Safety factor: $q \equiv \dfrac{d\varphi}{d\theta} = \dfrac{r}{R}\dfrac{B_t}{B_p}$ (number of turns magnetic field line goes around the toroidal direction when it goes 1 turn around the poloidal direction) is related to the stability of tokamak plasma.

MHD Stability Study by Energy Principle

MHD energy principle for fixed boundary mode:

$$\omega^2 \delta K(\vec{\xi}^*,\vec{\xi}) \equiv \omega^2 \int d^3x \rho \left|\vec{\xi}\right|^2 = \delta W(\vec{\xi}^*,\vec{\xi})$$

where $\delta W(\vec{\xi}^*,\vec{\xi}) = \int d^3x \left\{ \left| \delta\vec{B} + \frac{\vec{\xi}\bullet\nabla\psi}{\left|\nabla\psi\right|^2} \vec{J}\times\nabla\psi \right|^2 + \gamma P \left|\nabla\vec{\xi}\right|^2 \right.$

$$\left. - \left[\left(\frac{\vec{J}\bullet\vec{B}}{B^2}\right)^2 + \frac{\vec{J}\bullet\vec{B}}{B^2}\hat{S} + 2(\vec{\kappa}\bullet\nabla\psi)\frac{\partial P}{\partial\psi} \right] \left(\frac{\vec{\xi}\bullet\nabla\psi}{\left|\nabla\psi\right|}\right)^2 \right\}$$

$$\hat{S} \equiv -\frac{\vec{B}\times\nabla\psi}{\left|\nabla\psi\right|^2}\bullet\nabla\times\left(\frac{\vec{B}\times\nabla\psi}{\left|\nabla\psi\right|^2}\right) = \text{local magnetic shear}$$

Plasma unstable $(\omega^2 < 0)$ **energy sources :**

$$\left(\frac{\vec{J}\bullet\vec{B}}{B^2}\right)^2 + \frac{\vec{J}\bullet\vec{B}}{B^2}\hat{S} > 0 \quad \text{(free energy of parallel current-magnetic shear)}$$

$$(\vec{\kappa}\bullet\nabla\psi)\frac{\partial P}{\partial\psi} > 0 \quad \text{(free energy of pressure gradient in bad curvature)}$$

Before 1980s

- **Emphasis was on unstable MHD modes ($\omega^2 < 0$) by studying MHD energy principle**
- **Stable shear Alfven waves ($\omega^2 > 0$) are singular modes with frequencies forming continuous spectrum:** $\omega = k_{\|}(r)V_A(r)$
- **Collective effect caused by energetic particles (fast ions) are considered to be un-important in fusion plasmas !**
  - **Slowing-down energy distribution of energetic ions should be stable to velocity-space instabilities in uniform plasma**

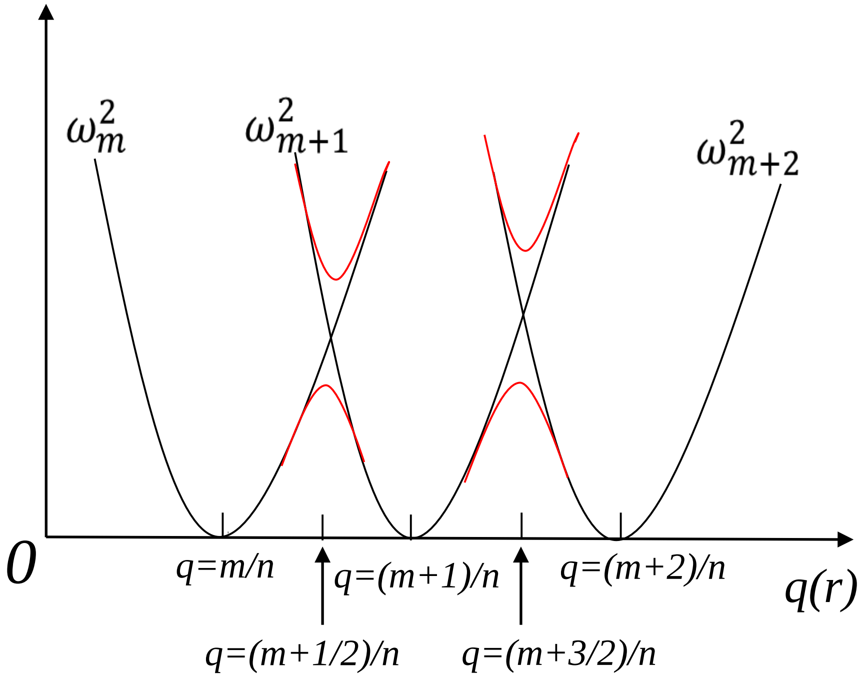
    Dawson, Furth, and Tenney, *PRL (1971)*

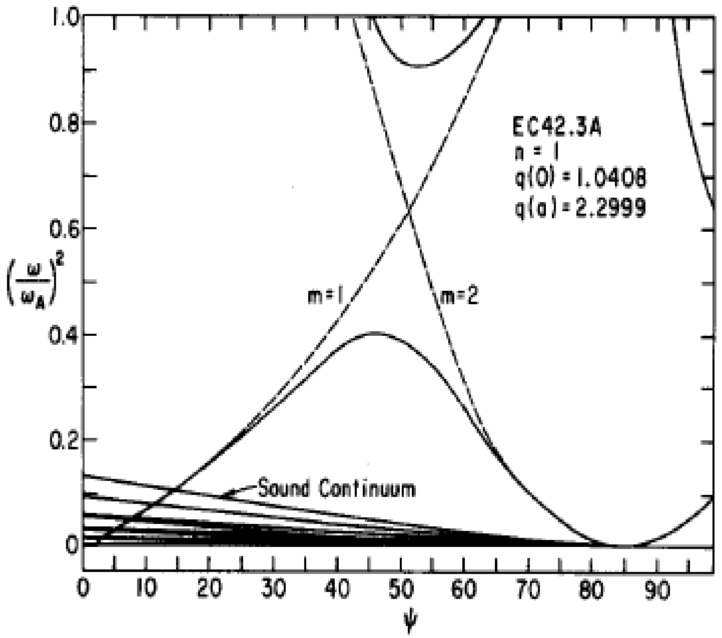
    Furth and Jassby, *PRL (1974)*
  - **Rosenbluth and Rutherford (*PRL (1975*)) considered possibility of destabilizing shear Alfven waves by pressure gradient of neutral beam injected ions. However, shear Alfven continuum modes are stable due to heavy continuum damping.**

After 1980 – beginning of fast ion physics

- Theoretical discovery of global TAEs (Cheng, Chen, Chance, 1985; Cheng, Chance, 1986)
- TAEs are cavity-like modes ubiquitous in toroidal plasmas, and do not suffer continuum damping
- Destabilization of TAEs by resonating with α-particles/fast ions (Cheng, Fu, Van Dam, 1989; Fu, Van Dam, 1989; Cheng, 1990).
- TAEs can cause anomalous fast ion loss
- TAEs have been observed in all major toroidal fusion devices (TFTR, DIII-D, NSTX, JT-60U, JET, ASDEX, LHD, K-STAR, etc.)
- A zoo of Alfven eigenmodes were uncovered from 1990s

Alfven Continuum & Gaps





Cheng & Chance, 1986

The shear Alfven continuum frequency for *m* is

$$\omega_m^2 = k_\parallel^2 V_A^2 = [(m - nq)V_A / qR]^2$$

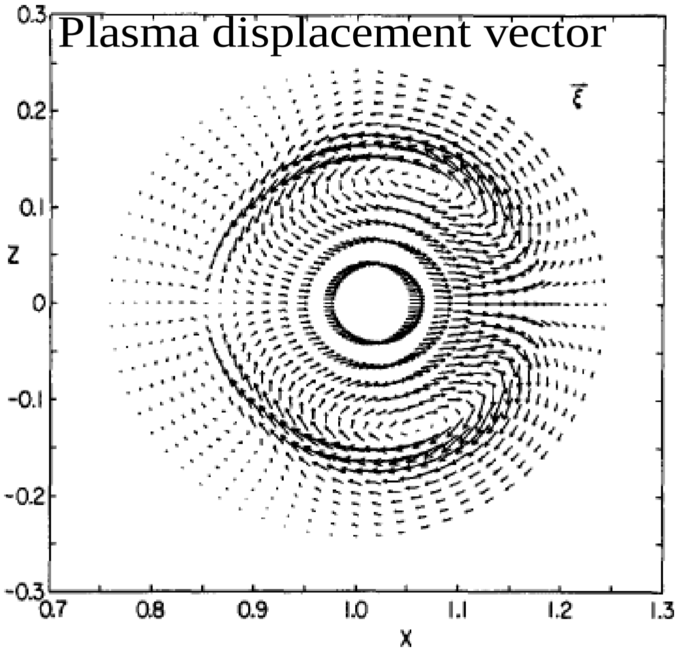
at $q = (m+1/2)/n, \quad \omega_m^2 = \omega_{m+1}^2 = (V_A/2qR)^2.$

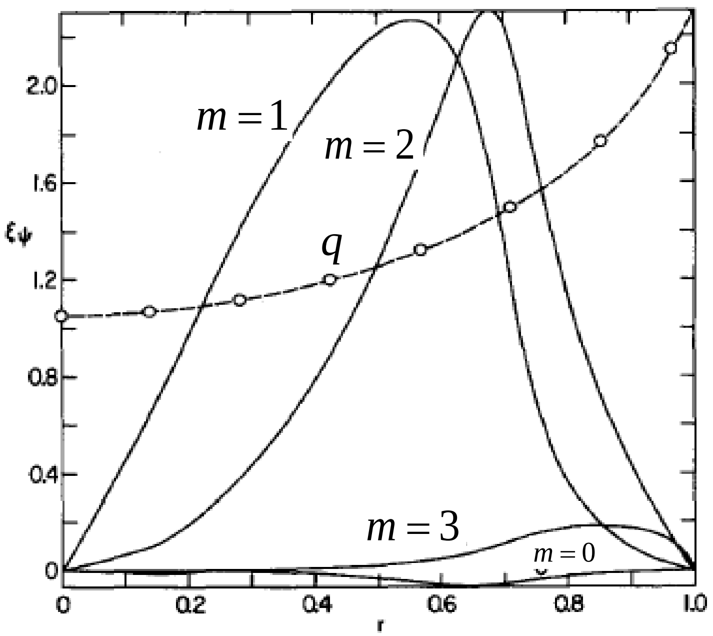
TAE continuum frequency gap at frequency crossing location:

$$\omega_\pm^2 = (V_A/2qR)^2[1 \pm \mathrm{O}(\varepsilon)]$$

n = 1 TAE computed by NOVA code

Cheng & Chance, 1986





- Existence of TAE with frequency inside the continuum gap (n=1 fixed boundary mode with $(\omega/\omega_A)^2=0.5$, $\omega_A= V_A(0)/q(0)R$)
- TAEs can exist for all toroidal-*n* modes (cavity-type modes)
- For each *n*, there can be more than one TAE with different poloidal mode structures

High-*n* TAEs

- In a zero-β, large aspect ratio tokamak plasma, **high-*n* (*n* >> 1) shear Alfven waves** can be modeled by

$$\left[\frac{d^2}{d\theta^2}+\left(\frac{\omega}{\omega_A}\right)^2(1-2\varepsilon\cos\theta)-\frac{s^2}{(1+s^2\theta^2)^2}\right]\Phi=0$$

$\varepsilon = r/R$, $s = rq'/q$, $\omega_A = V_A/qR$

- For s = 0, singular waves are described by the **Mathiu equation**; waves move in **a periodic potential well**, similar to electrons moving in a periodic lattice in solid state physics

  ➔ continuous frequency bands (energy bands) and gaps

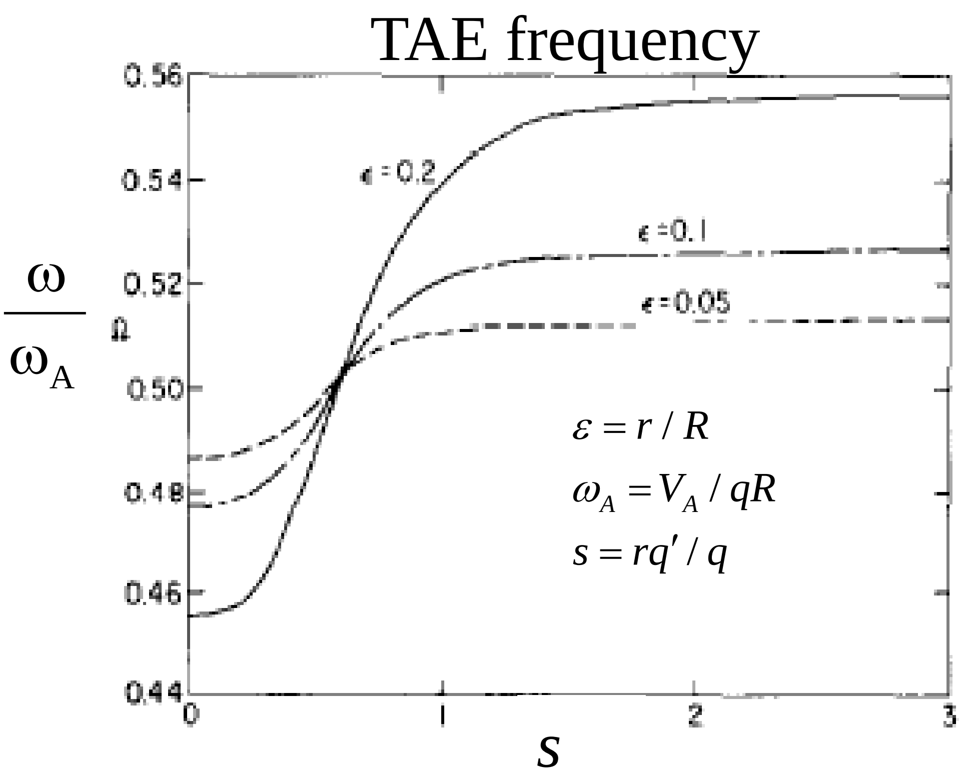
- There is an infinite number of frequency gaps centered at

  $\omega \sim j\omega_A/2$, $j = 1, 2, \ldots.$

- The lowest continuum gap is bounded by $\omega_\pm^2 = (1 \pm \varepsilon)\omega_A^2/4$

Cheng, Chen, Chance, 1985

High-*n* TAEs



For s << 1, $$\frac{\omega}{\omega_A} \simeq \pm\frac{1}{2}\left[1-\frac{\varepsilon}{2}\left(1-s^2\pi^2/8\right)\right]$$

For s >> 1, $$\frac{\omega}{\omega_A} \simeq \pm\frac{1}{2}\left[1+\frac{\varepsilon}{2}\left(1-\pi^2/72s^4\right)\right]$$

- **For s ≠ 0, periodicity in the wave potential is broken, similar to periodicity breaking by impurity atoms or other effects.**
- **TAEs are similar to discrete electron energy states in aperiodic lattice due to periodicity breaking in solid state physics.**
- **TAEs can exist for all toroidal *n*-mode numbers.**

Cheng, Chen, Chance, 1985

TAE Instability

- **TAEs can exist for all toroidal mode numbers (*n*).**
- **Fast ions resonate with TAEs if $v_h \sim V_A$.**

$v_h > 0.5\ V_A$ can be satisfied for α-particles, MeV protons in ICRH operation, and MeV N-NBI Deuterium ions.

- **Necessary condition for fast ion instability drive:**

Free energy in fast ion pressure gradient overcomes velocity space damping effect if $nq(v_h/V_A) > (r/R)(L_h/\rho_h)$.

- **Sufficient condition for TAE instability:**

γ_h (fast ion drive) > γ_d (thermal plasma damping)

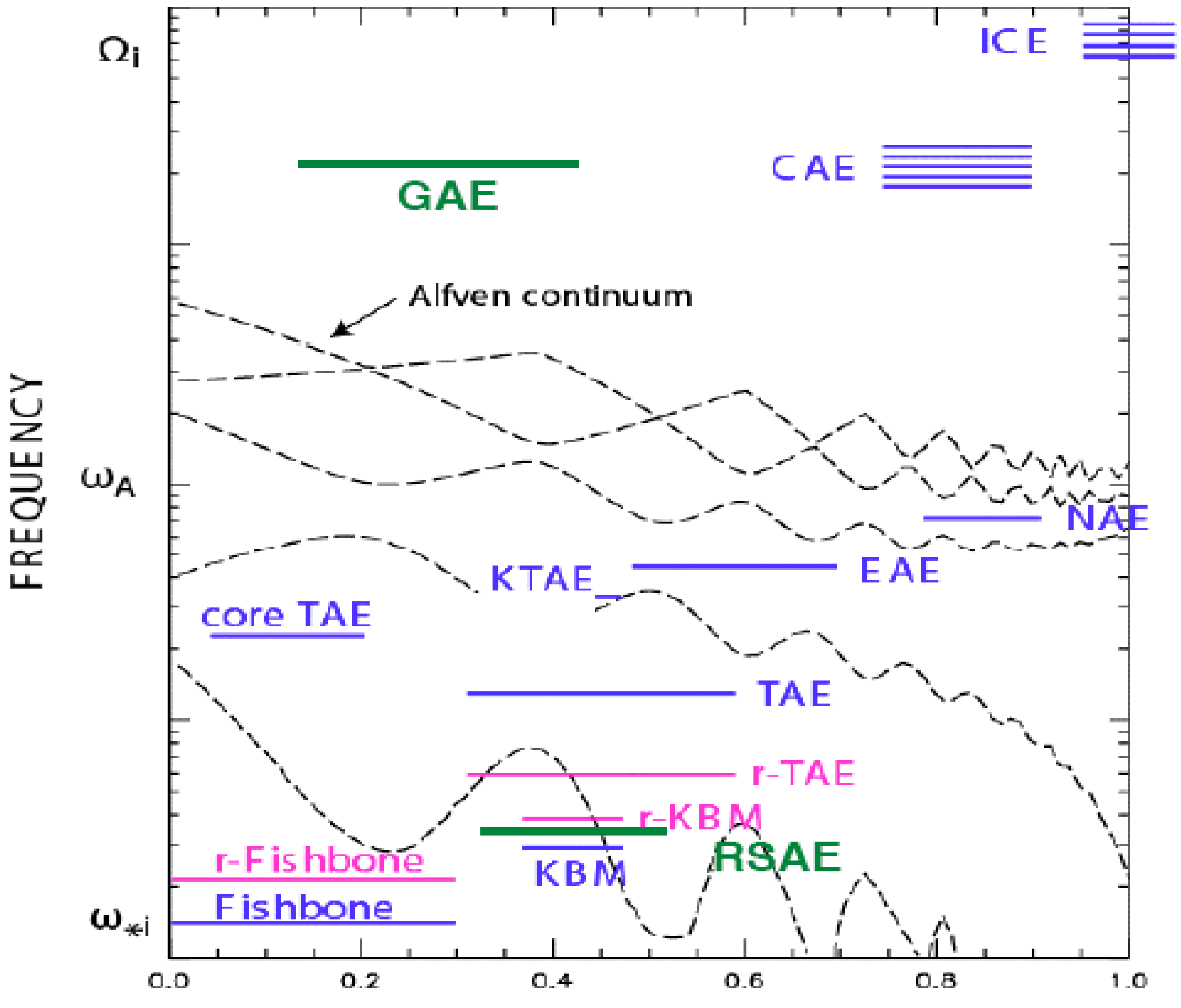
- **Multiple TAEs are expected to be robustly unstable in burning plasmas !!**

NOVA-K code (Cheng, 1990; 1993) was developed to calculate stability of TAEs in tokamaks !

Observations of TAEs in Major Tokamaks

- First experimental observation of TAEs in TFTR (Wong et al., 1991)
- TAEs observed in DIII-D (Heidbrink et al., 1991; Strait et al., 1993)
- Alpha-particle driven TAEs in TFTR DT-plasma with small amount of α-particles (Nazikian et al., 1997)
- TAEs observed in JET (Ali-Arshad & Campbell, 1995; Sharapov et al., 2001)
- Bursting TAEs in JT-60U by NNB and significant fast ion loss (Kimura et al., 1998; Shinohara et al., 2002; Ishikawa et al., 2005)
- Fast ion loss by bursting TAEs in NSTX (Fredrickson et al., 2003)
- …….

A Zoo of Alfven Eigenmodes



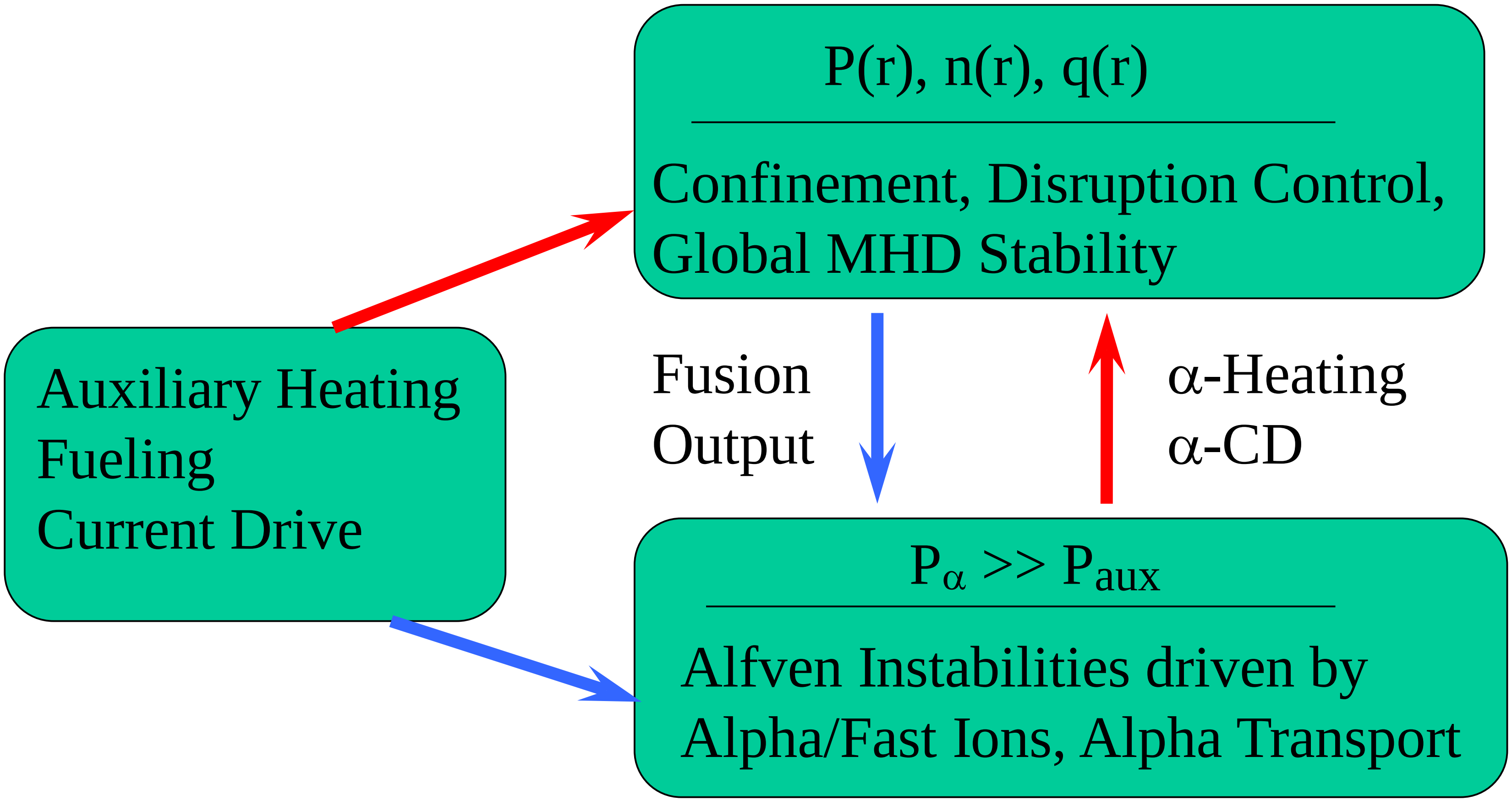
Heidbrink, Phys. Pl. 9 (2002) 2113

- **TAEs are generic issue for all toroidal fusion devices**
- **TAEs are expected to be most serious in fast ion transport**

Fast Ions in D-T Burning Plasmas

- **In burning plasmas, 3.5 MeV α-particles are dominant heating source over auxiliary heating sources (NBI, NNB, ICRF, ECRH, etc.): $P_\alpha >> P_{aux}$**
- **Alpha particle heating controls thermal plasma profiles, global plasma stability and confinement.**
- **Alfven instabilities can cause anomalous loss of alpha/fast ions.**
- **Significant loss of alpha/fast ions degrades plasma heating and current drive efficiency in burning plasmas, and can even quench D-T burning.**
- **Lost alpha/fast ions tend to localize near outer mid-plane and can cause localized damage on first wall of fusion reactors.**

Summary on TAEs
& Burning Plasmas Physics



α-particle interaction with thermal plasmas is strongly nonlinear process.
Must develop integrated α-physics to understand burn plasma behaviors !

Solar Flares & Coronal Mass Ejections

Sun is the most energetic particle accelerator in the solar system:

- Ions up to ~ 10s of GeV

- Electrons up to ~ 100s of MeV

Acceleration to these energies occurs in two processes:

- ***Large Solar Flares*** **are most powerful explosion in the solar system, release up to ~ *10^{32} - 10^{33} ergs*, and accelerate particles to emit radio waves, EUV, X-ray, γ-ray**
- ***Fast Coronal Mass Ejections (CMEs)*** **carry 1-10 billion tons of plasma at V_{cme} = 1000-2000 km/s, growing into a cloud tens of millions of miles wide, creating shock waves to accelerate charged particles to ultra-high energy**

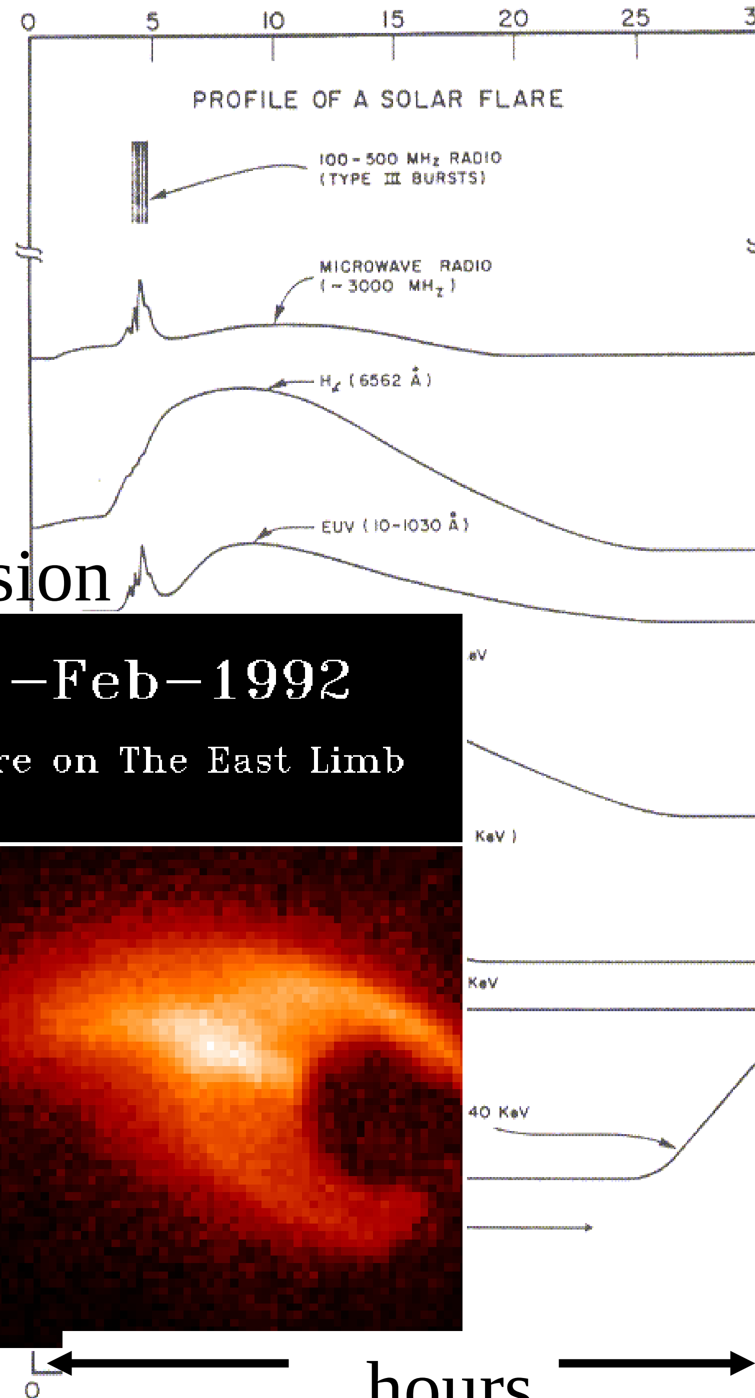
Solar Flares

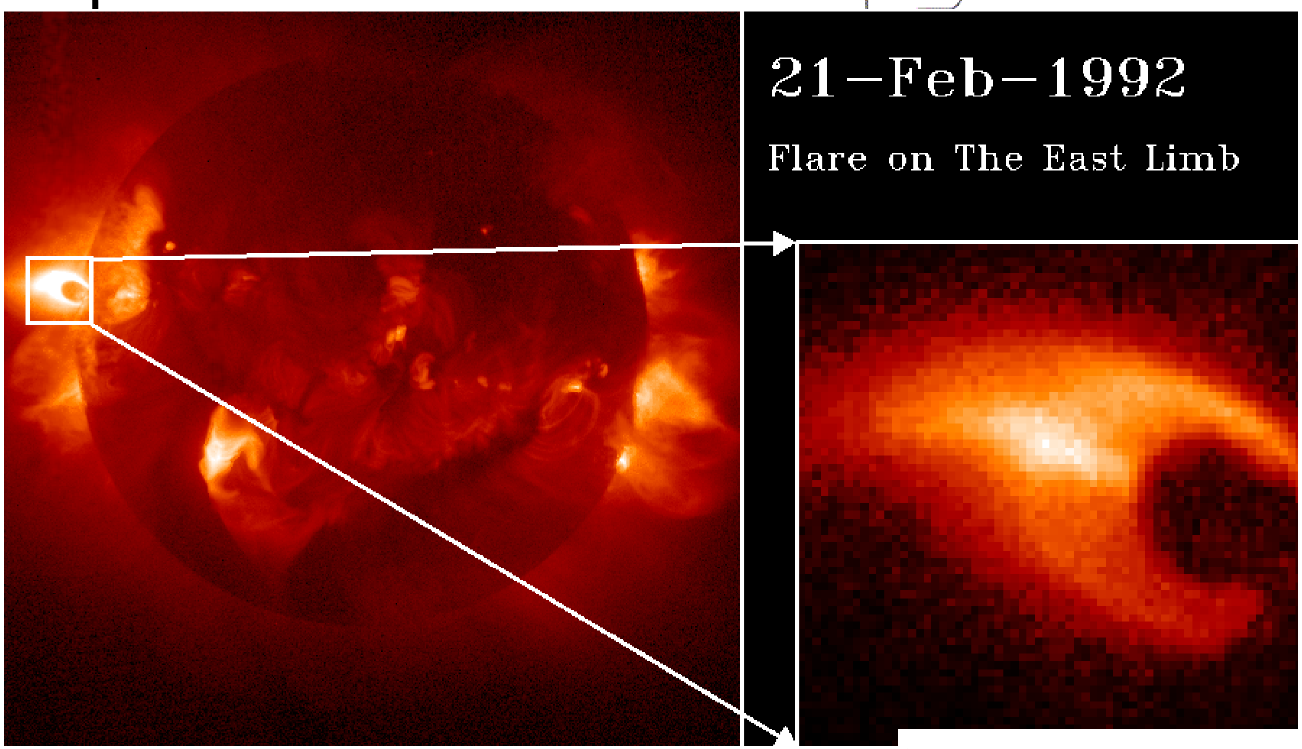
Time scale: 10 min. – 1 day

Energy: $10^{27} – 10^{33}$ ergs

Spatial Size: $10^4 – 10^5$ km

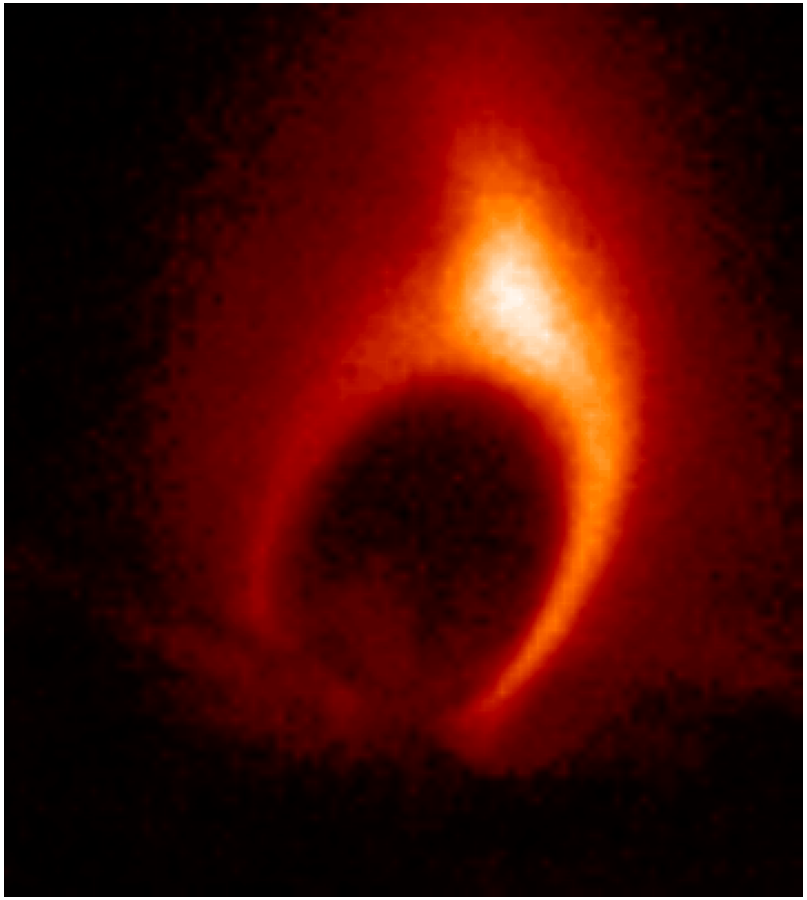
Impulsive nonthermal emission



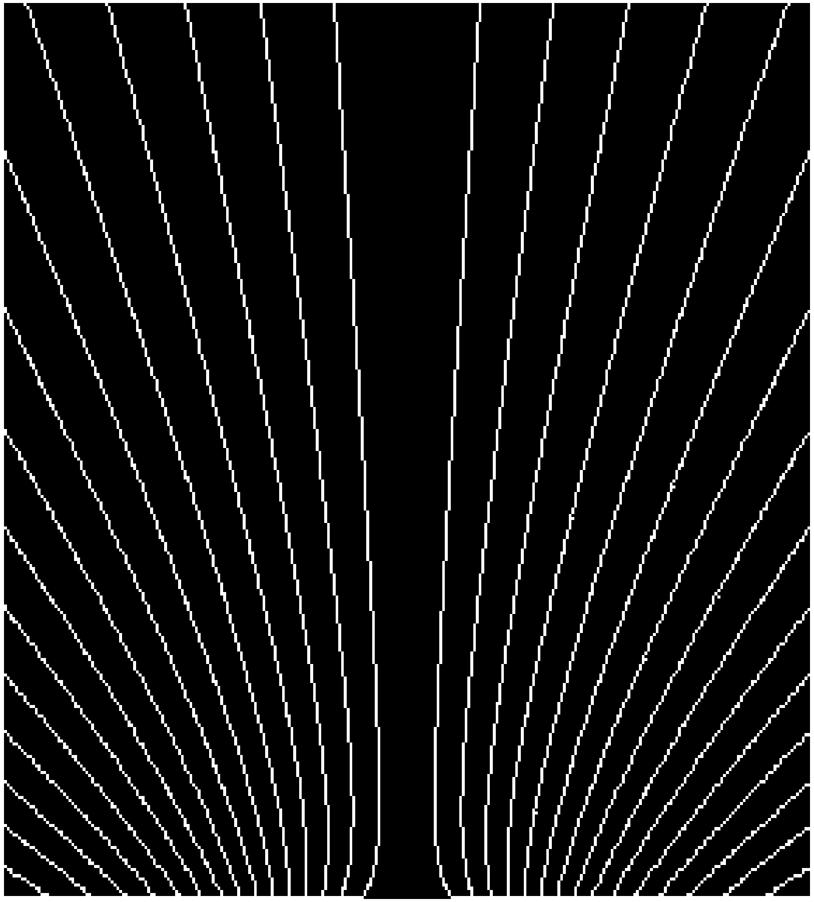


SXT on Yohkoh satellite

Magnetic Reconnection Drives Solar Flares

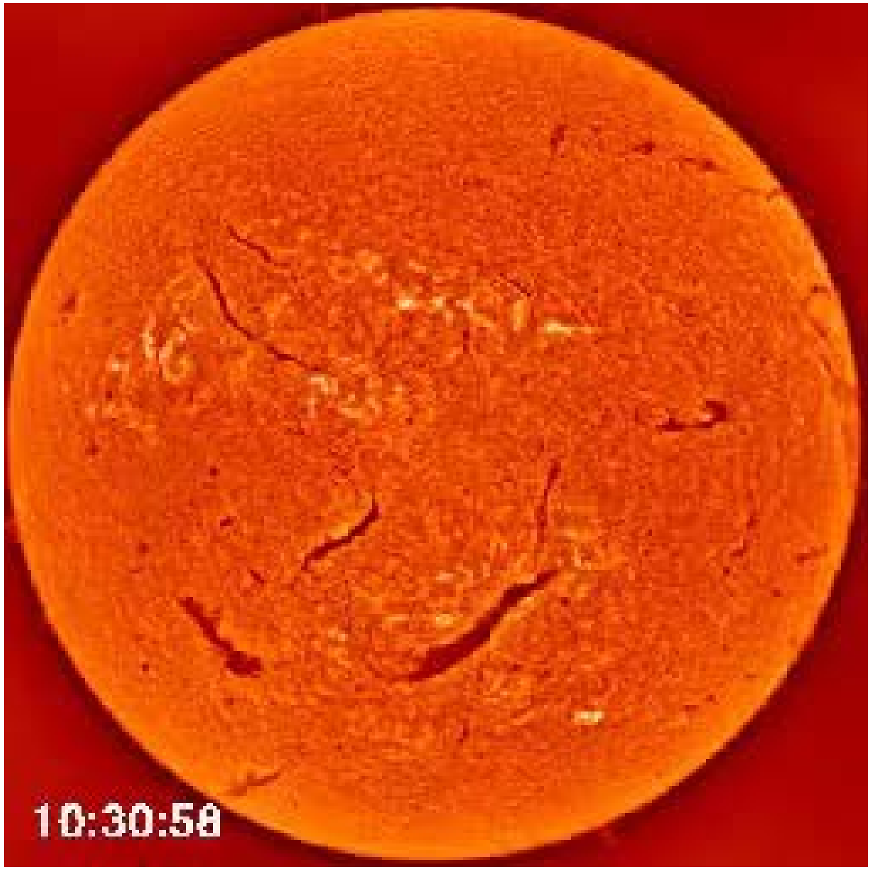


Yokoh SXR flare obs.



Magnetic reconnection

Flare Signatures:
H_α Filament Eruption & H_α Ribbon Expansion



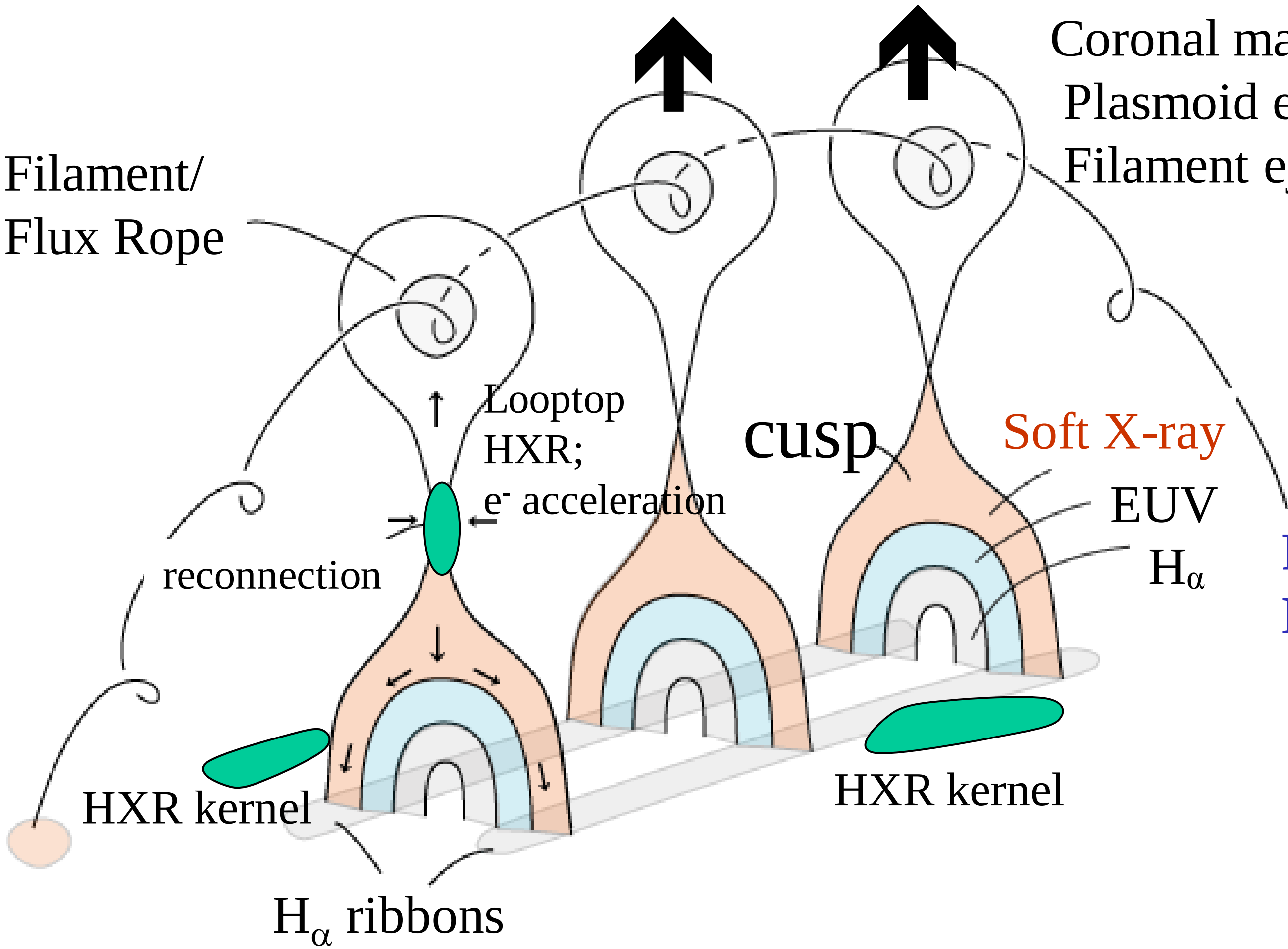
H_α Image of two-ribbon flares & filament eruption

- **H_α filament eruption is CME observed in lower corona**
- **Expanding H_α ribbons are flare signatures on solar surface**

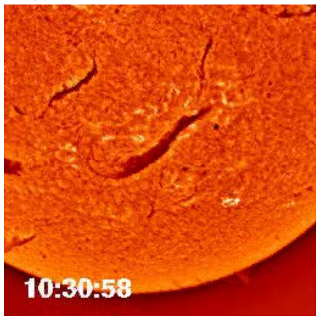
CMEs are accompanied with Large Flares (movie provided by NASA)

- **CME and flare are manifestations of same magnetic reconnection process**
- **CME is accompanied by flare.**

Magnetic Reconnection Model of Solar Flares and Coronal Mass Ejections



Filament eruption & H_α ribbon expansion



Particle Acceleration/Heating in Driven Magnetic Reconnection

- **Particles gain energy by E-field acceleration**
- **Electric field sources**
  - **Reconnection E-field E_{rec} : due to E-field in merging field lines and inductive conversion from magnetic field**
  - **Electrostatic E-field E_{es} : due to charge separation**
  - **E-fields in waves**
- **PIC simulation results**
  - **E_{rec} and E_{es} combine to form $E_{||}$ due to inductive quadrupole B-field production**
  - **E_{rec} and E_{es} and $E_{||}$ are main causes of particle acceleration/heating**
  - **Wave E-fields are secondary**

Magnetic Reconnection Rate (or E_{rec})

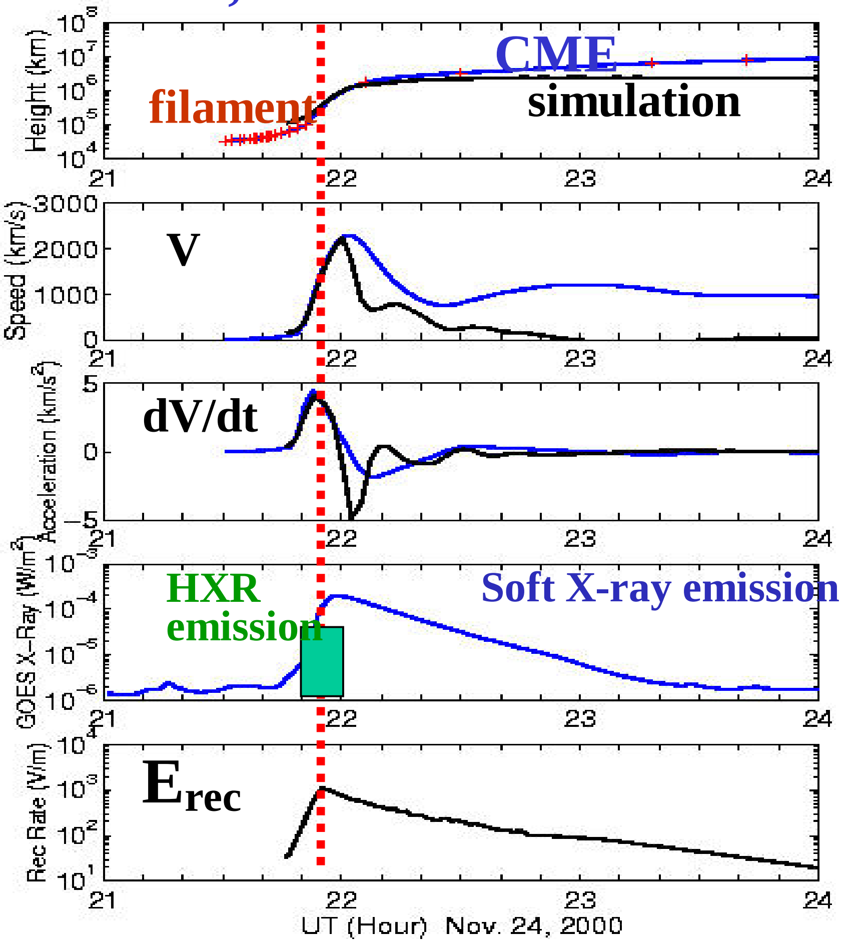
- **Internal processes in reconnection region:**
  - **Current sheet dissipation in Sweet-Parker model:**

$$E_{rec} = \left(\sqrt{2}\eta / LV_{Ai}\right)^{1/2} V_{Ai} B_0$$

    ➔ **require anomalous resistivity η_{an} to get realistic E_{rec}**
  - **Slow mode shock in Petschek model: $E_{rec} \sim 0.1\ B_0 V_A$**
  - **Collisionless kinetic processes: $E_{rec} \sim 0.1\ B_0 V_A$**
- **External processes:**
  - **E_{rec} is due to E_{drive} in merging field lines: $E_{rec} \sim E_{drive}$, which varies over a wide range of values as observed in space plasmas**
  - **E_{rec} is enhanced by flux rope (CME) acceleration which causes enhanced thinning of current sheet**

Prediction of E_{rec} from MHD Simulations and Flare-CME Observations

Nov. 24, 2000 X1.8 flare-CME event



Prediction of E_{rec} ~ O(1kV/m) for X-class flares in MHD magnetic reconnection simulations with anomalous resistivity.

Choe, Cheng, *ApJ*. (2000)

Cheng, Ren, Choe, Moon, *ApJ*. (2003)

- **Fast magnetic Reconnection coincides with acceleration of CME motion & impulsive flare HXR emission during flare rise phase**
- **Good agreement between MHD simulation results (black curves) and CME observation (blue curves)**
- **Peak reconnection electric field at X-line is E_{rec} ~ 1 kV/m for this X-class flares**

Results suggest fast magnetic reconnection is induced by acceleration of CME velocity

E_{rec} Determination from Two-ribbon Expansion and CME Acceleration

Event (magnitude)	2000/09/12 (M1.0)	2001/10/19 (X1.6)
Max. CME acceleration (km/s^2)	0.2 - 0.4	2.
Duration of acceleration (min)	> 120	< 40
Mean and Max. velocity (km/s)	1550 & 1700	900 & 1450
Max. magnetic field (Gauss)	200	1200
Max. electric field (V/m)	50	580
Duration of reconnection (min)	> 120	~ 30

J. Qiu, H. Wang, C. Z. Cheng and D. E. Gary, *Astrophys. J.*, 604, 900 (2004)

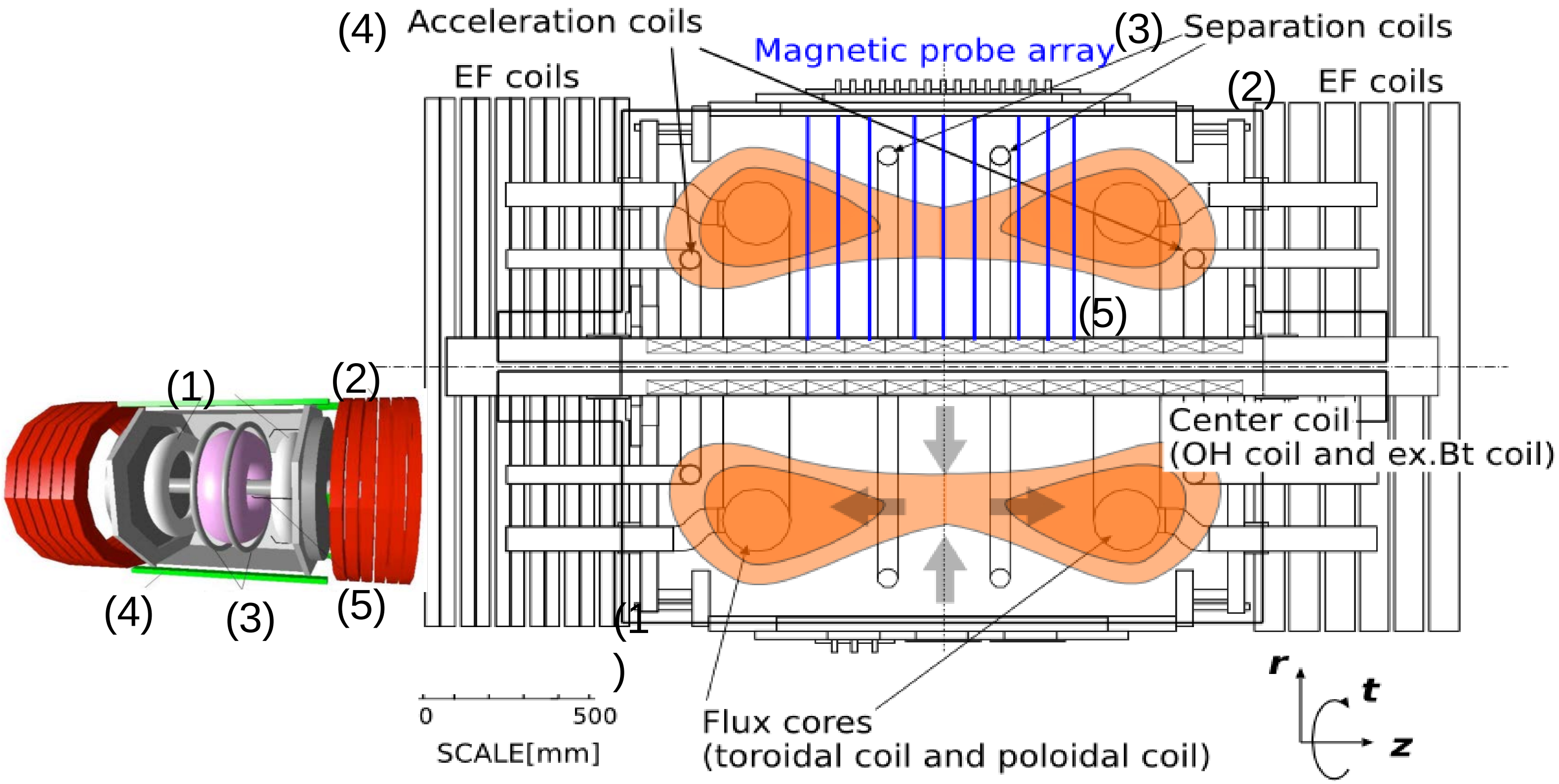
Peak E_{rec} Measurements from Different Solar Emissions

2003/10/29 X10 flare

	Peak E_{rec} (kV/m)	Formula	Instrument
Xu et al. (2004)	4.5	$V_\perp B_n$	Near-Infrared
Jing et al. (2005)	3.8	$V_\perp B_n$	H_α
Krucker et al. (2005)	6.7	$V_\perp B_n$	RHESSI HXR
Liu & Wang (2009)	1.7	$\frac{1}{L}\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{a}$	H_α
Yang et al. (2011)	6.0	$V_\perp B_n$	RHESSI HXR
Yang et al. (2011)	2.6	$\frac{1}{L}\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{a}$	TRACE UV

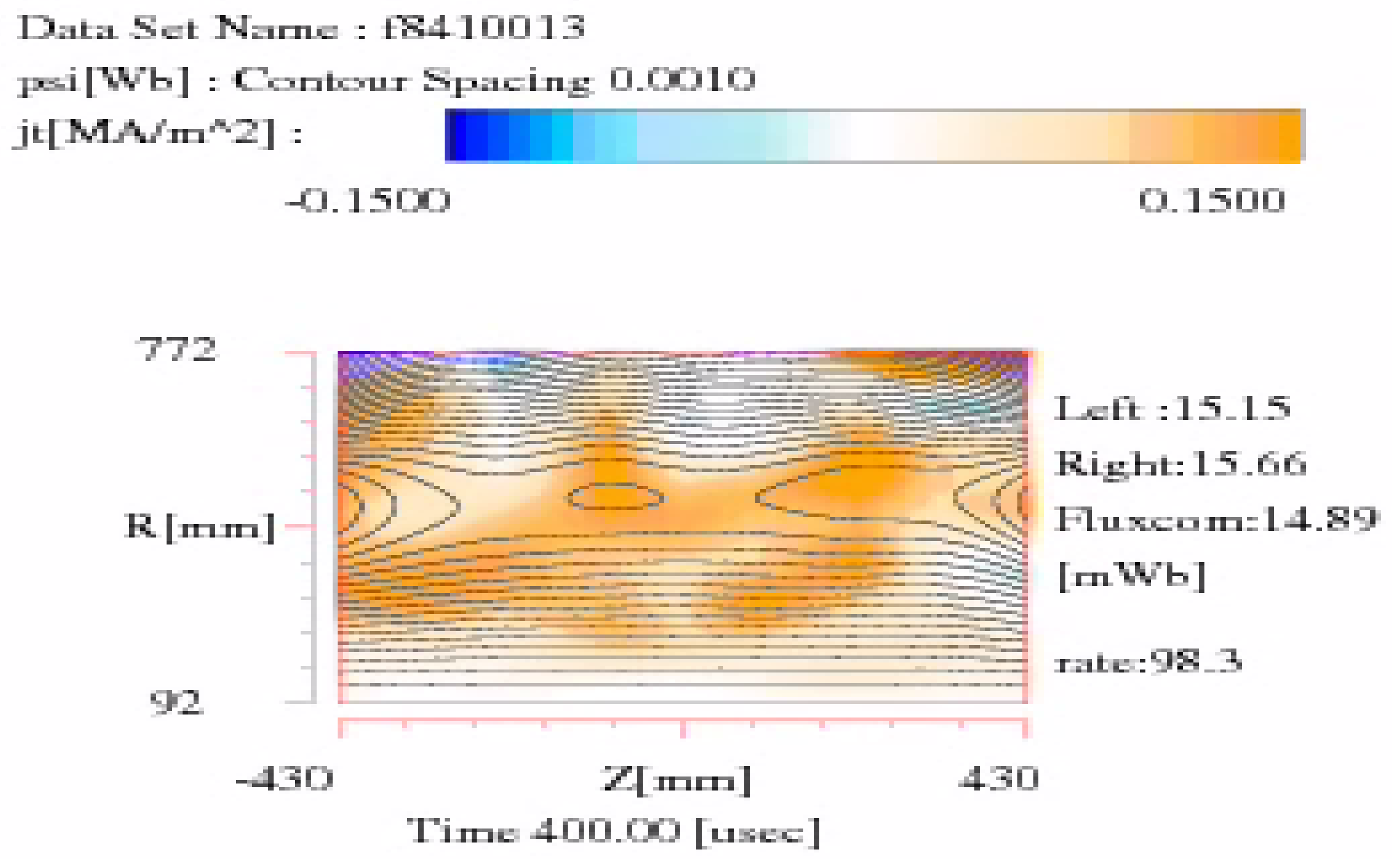
Y. H. Yang, C. Z. Cheng, S. Krucker and M. S. Hsieh, Astrophys. J. 732, 15 (2011)

Experimental demonstration of impulsively fast reconnection and acceleration of plasmoid motion at University of Tokyo (Y. Ono et al.)



Vertical cross-section of **TS-4 merging device**: two flux cores and separation coils are used to control magnetic reconnection from common flux to private flux.

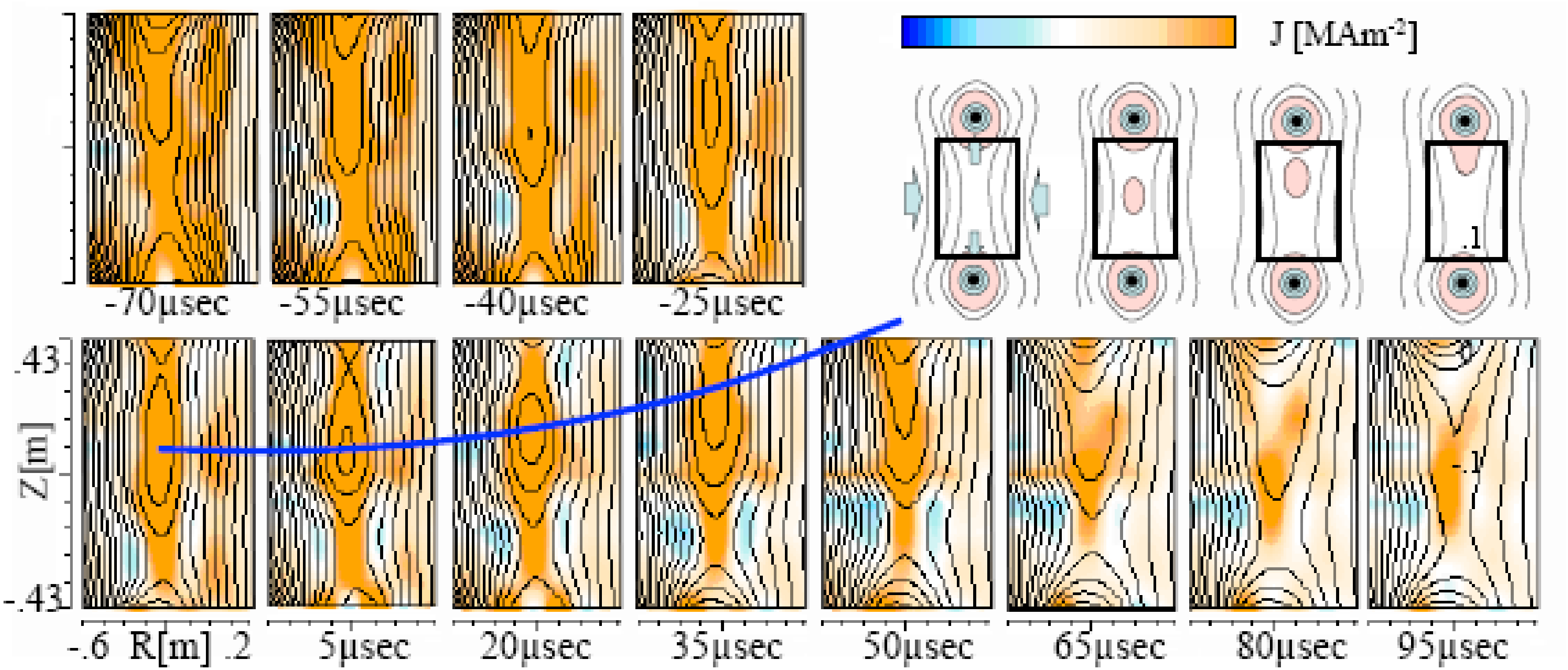
TS-4 Plasma Merging Experiment



Provided by Y. Ono

High external inflow causes current sheet to eject a plasmoid, increasing reconnection speed impulsively.

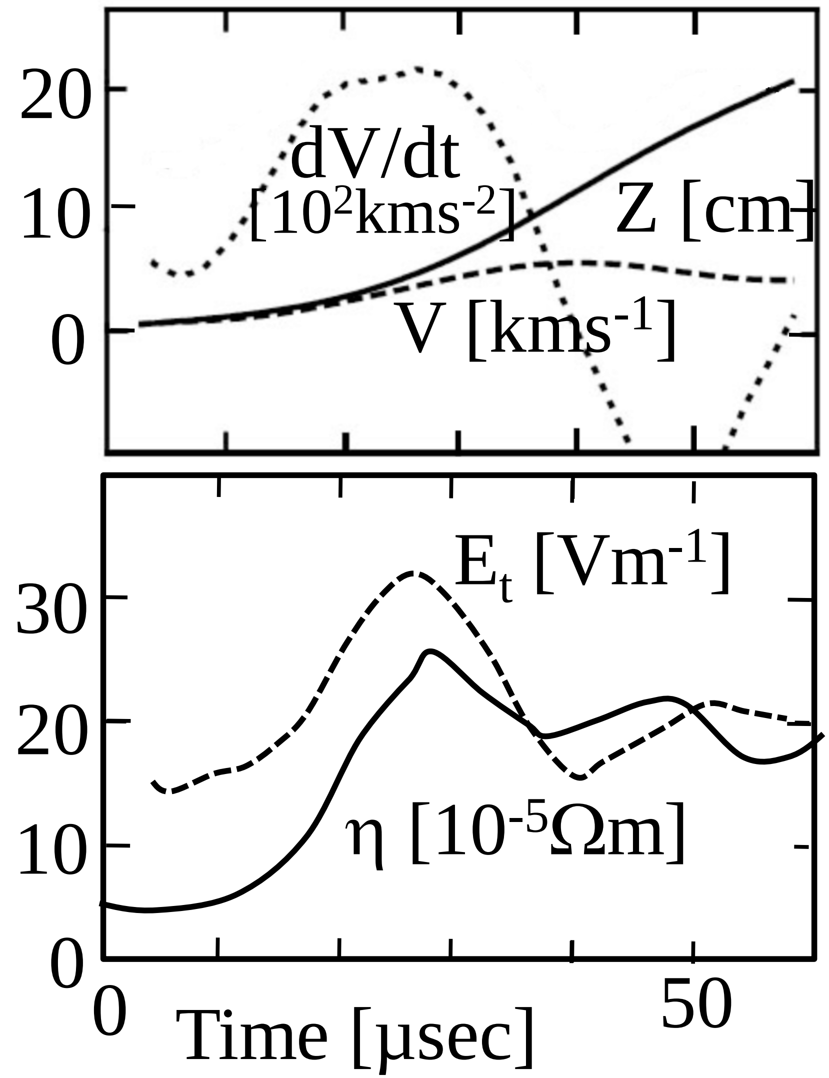
Plasmoid Ejection



Poloidal flux contours with toroidal current density j_t (red and blue colors) under high inflow condition.

TS-4 Plasma Merging Experiment

Anomalously fast resistivity η & reconnection electric field E_t are enhanced simultaneously (impulsively) during acceleration of plasmoid ejection motion



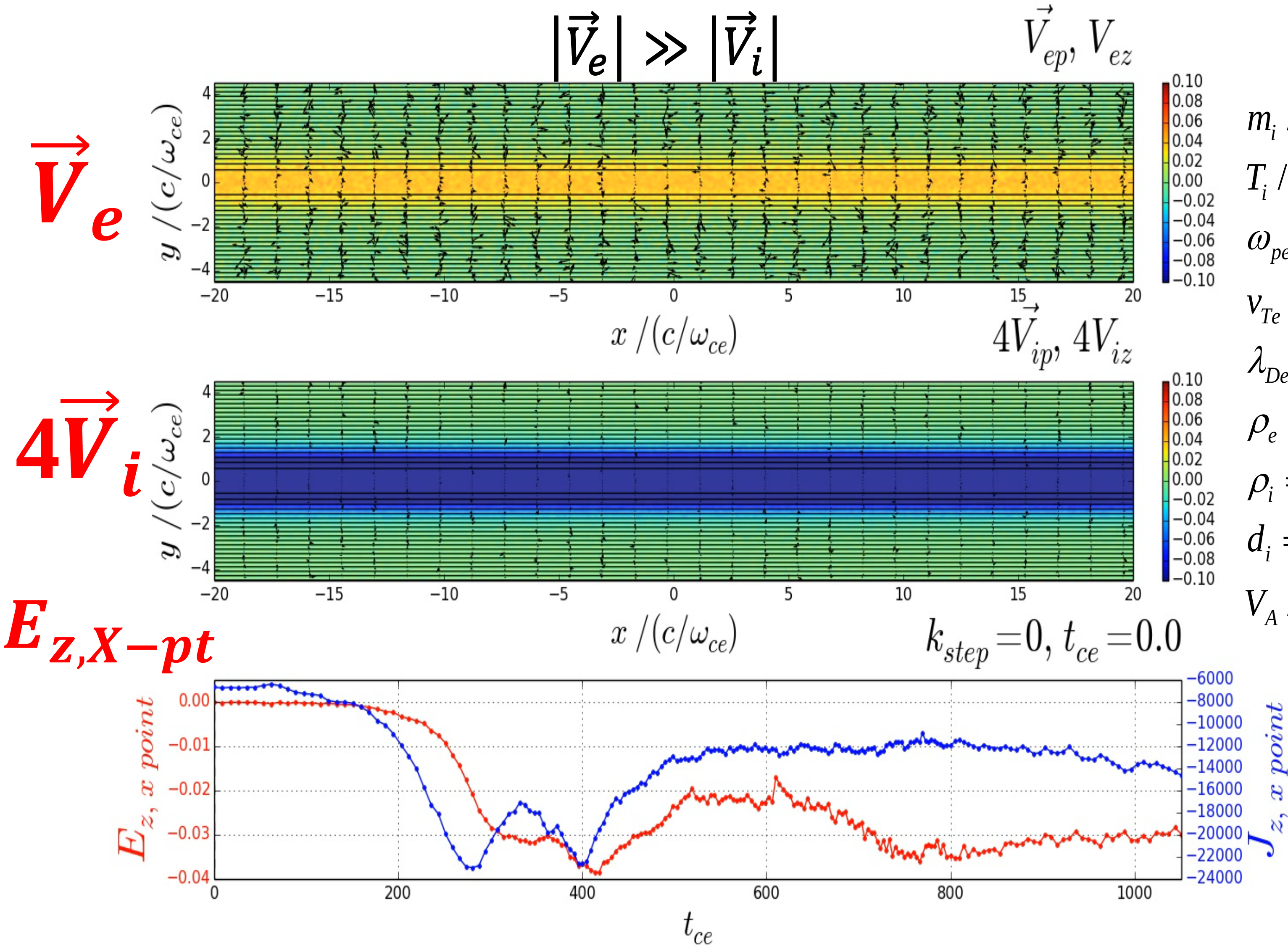
Provided by Y. Ono

Particle-in-Cell Simulation of Driven Magnetic Reconnection

- **Breakdown of MHD or fluid models in entire reconnection boundary layer: from upstream separatrix regions to reconnection region to downstream region**
- **Decoupling of electron & ion dynamics leads to charge separation and creates electrostatic electric field $\vec{E}_{es}$**
- **Generation of out-of-plane B-field (quadrupole structure) B_z , which combines with driving E-field E_z to produce parallel E-field $\vec{E}_{||}$**
- **Particle heating/acceleration by $\vec{E}_{||}$, $\vec{E}_{es}$ and $\vec{E}_z$**

1. C. Z. Cheng et al., Phys. Plasmas 22, 101205 (2015)
2. S. Inoue et al., Nuclear Fusion 55, 083014 (2015)
3. C. Z. Cheng et al., Plasma Fusion Res. 11, 1401081 (2016)

Anti-parallel Driven Magnetic Reconnection: Electron and Ion Flows (V_z in color)



$m_i / m_e = 100$

$T_i / T_e = 1$

$\omega_{pe} / \omega_{ce} = 4$

$v_{Te} / c \approx 0.14$

$\lambda_{De} \approx 0.0354\ c / \omega_{ce}$

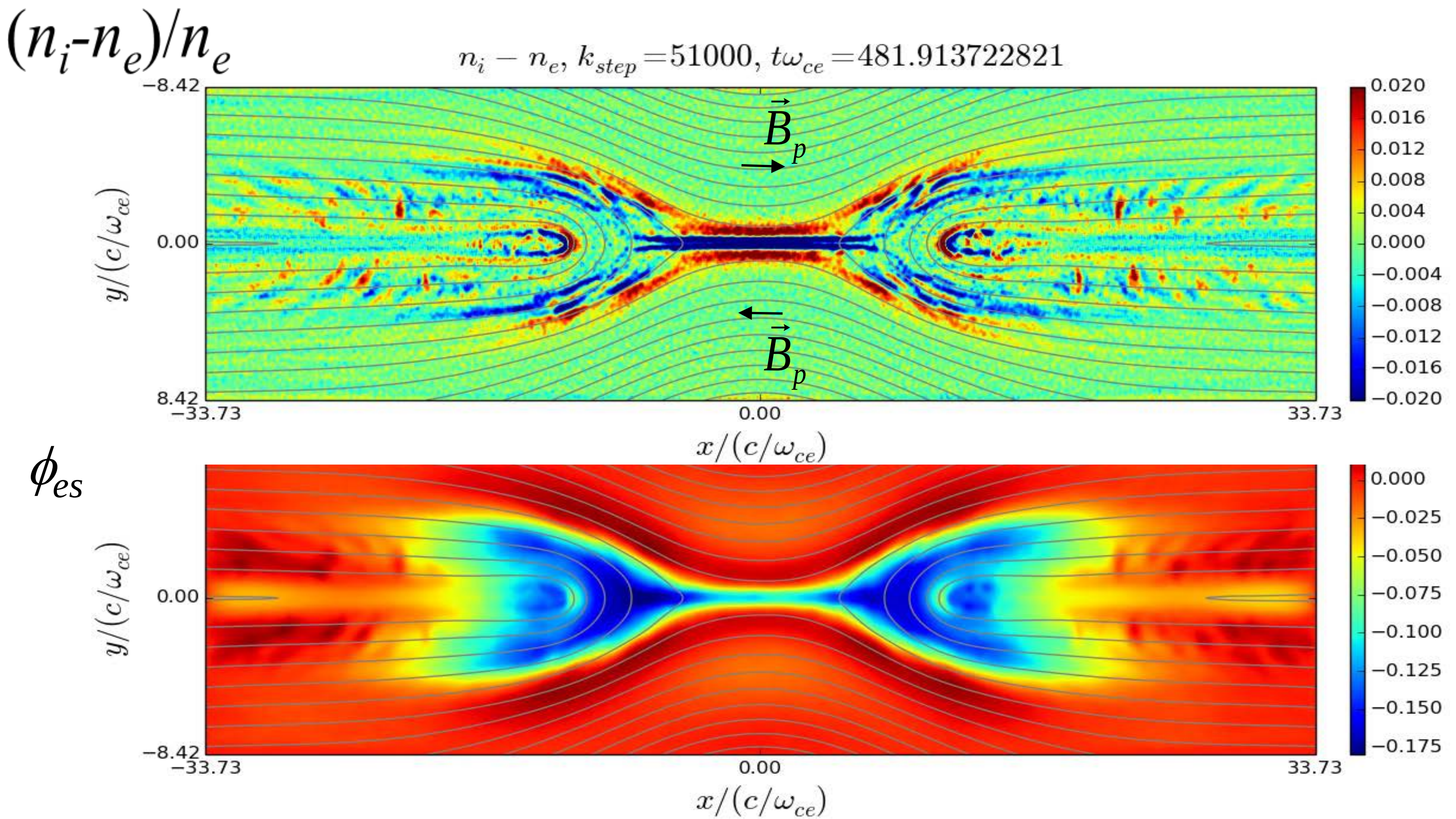
$\rho_e = 0.14\ c / \omega_{ce}$

$\rho_i = \mathbf{1.4}\ c / \omega_{ce}$

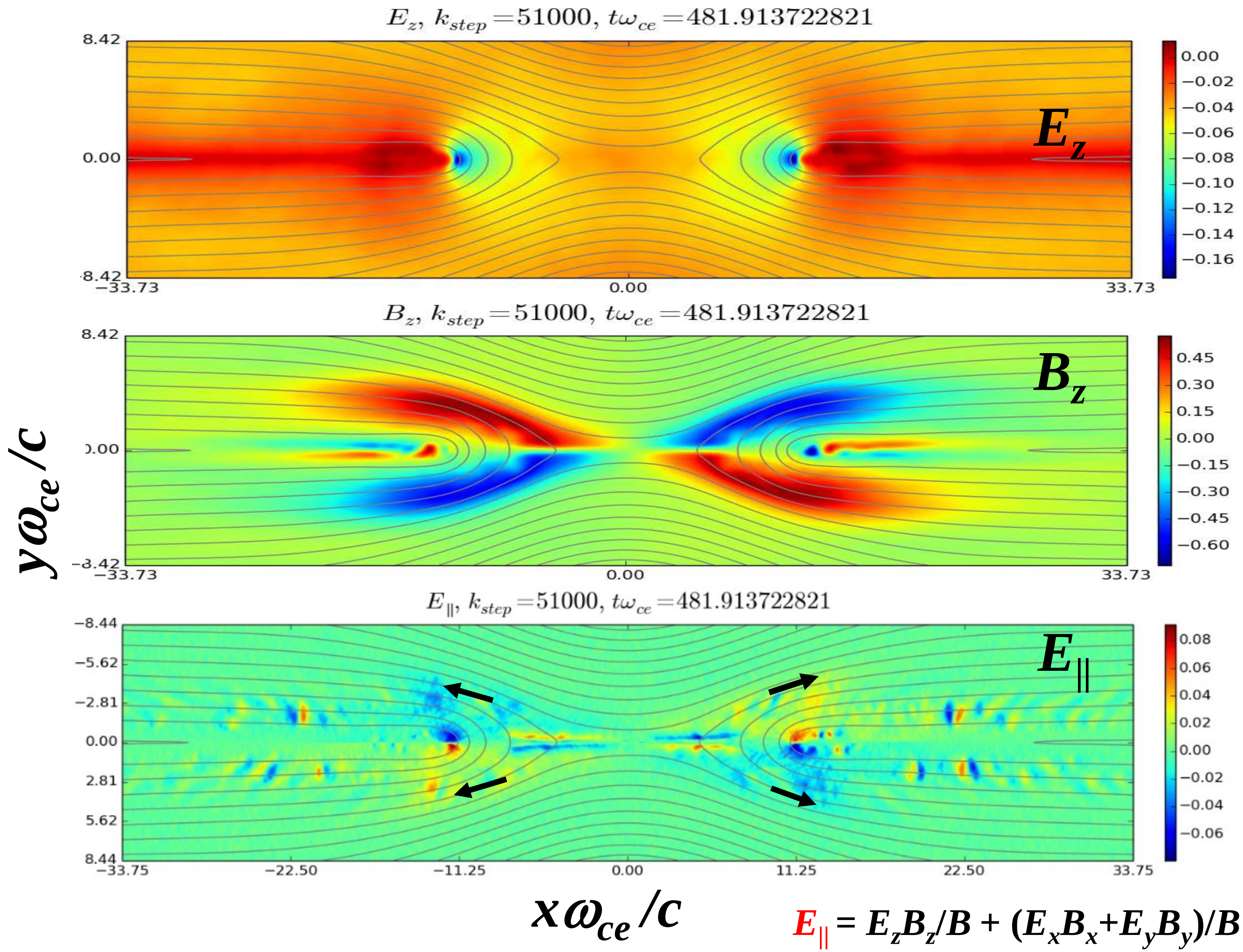
$d_i = c / \omega_{pi} \approx \mathbf{2.46}\ c / \omega_{ce}$

$V_A / c = \mathbf{0.025}$

Charge Separation & Electrostatic Potential

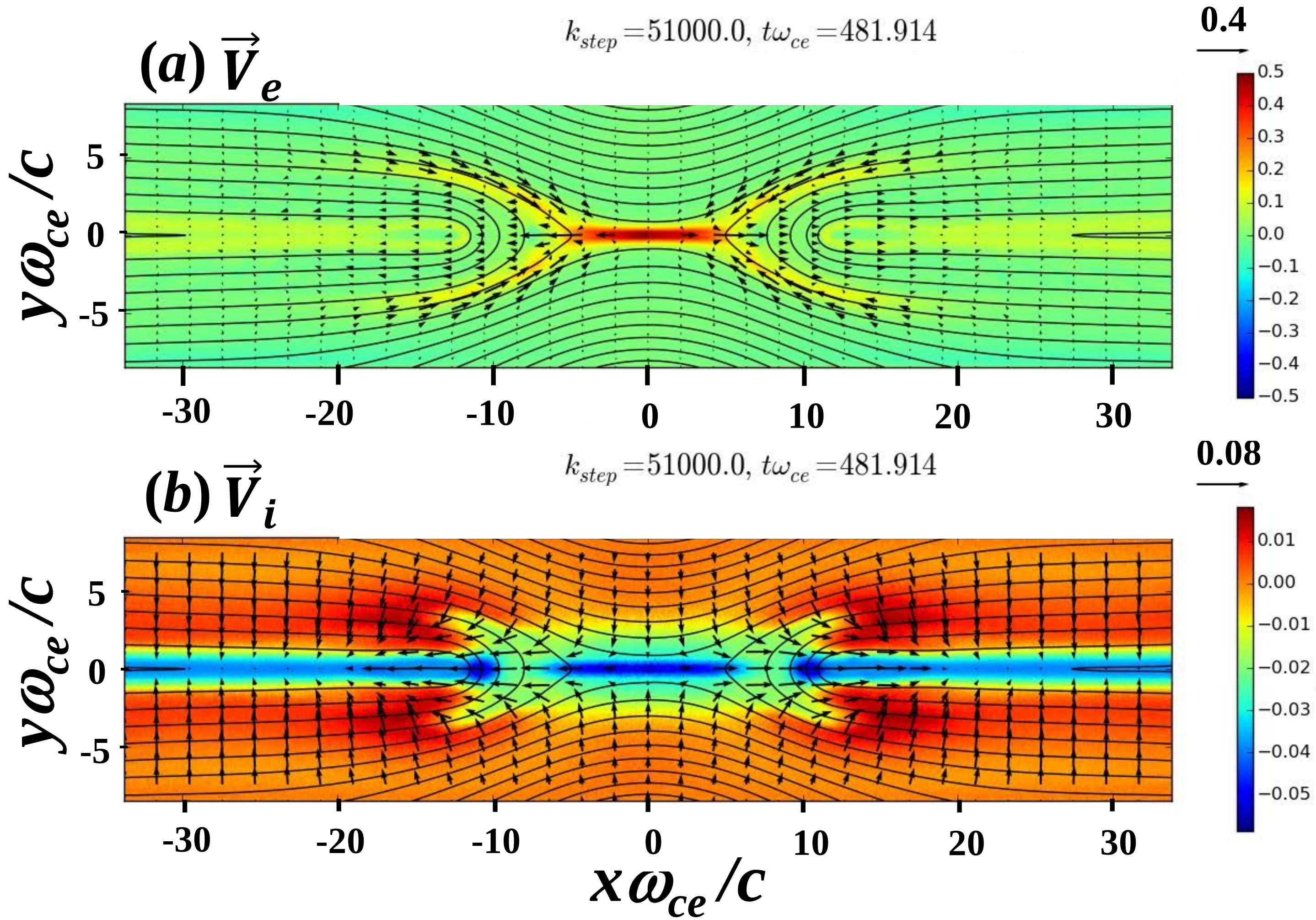


Generation of Parallel E-field $E_{\parallel}$



$E_{\parallel} = E_z B_z/B + (E_x B_x + E_y B_y)/B$

Different Electron and Ion Flow Patterns

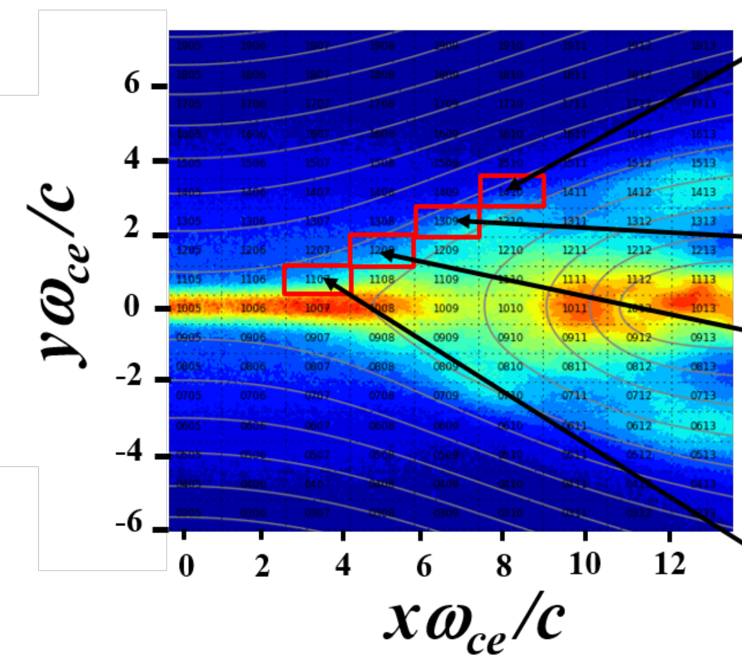


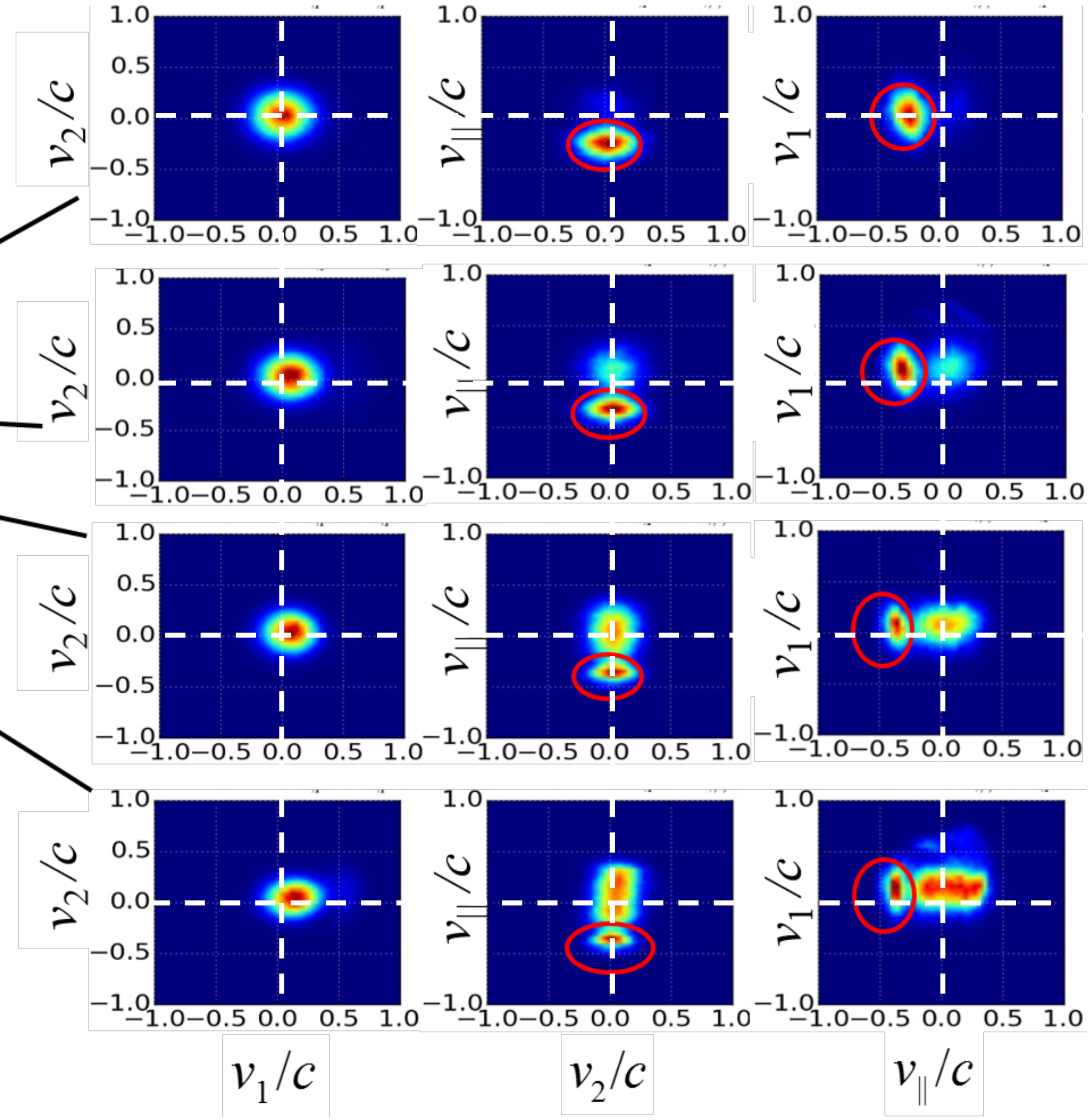
Electron Acceleration/Heating

- **Electron outflow speed is >> ion outflow speed; poloidal current J_p flows toward reconnection region** and produces **quadrupole B_z field**.
- **Parallel electric field** $\vec{E}_{||} = (\vec{E}_{es} \cdot \vec{B}_p + E_z B_z)/\boldsymbol{B}$ **is produced around searatrix region** and downstream mainly by $E_z B_z/\boldsymbol{B}$
- **Electrons are accelerated by $E_{||}$ around separatrix and flow mainly along separatrix field lines toward reconnection region**.
- **Accelerated electrons flow mainly through reconnection region to downstream and also across separatrix with merging field lines.**
- **In reconnection region** electrons also **gain energy by driving E_{rec}** in **electron meandering region** and by bipolar electrostatic field $\vec{E}_{es}$ **via $\vec{v}_{ez} = \vec{E}_{es} \mathrm{x} \vec{B}_x / B^2$ drift**.
- **Outflowing electrons have super-Alfvenic speed** and **are thermalized in downstream by stronger magnetic field.**

Electron Acceleration by $E_{\parallel}$ along B around Separatrix Regions ($B_g = 0$)

$$f_e(v_1, v_2, v_\parallel)$$



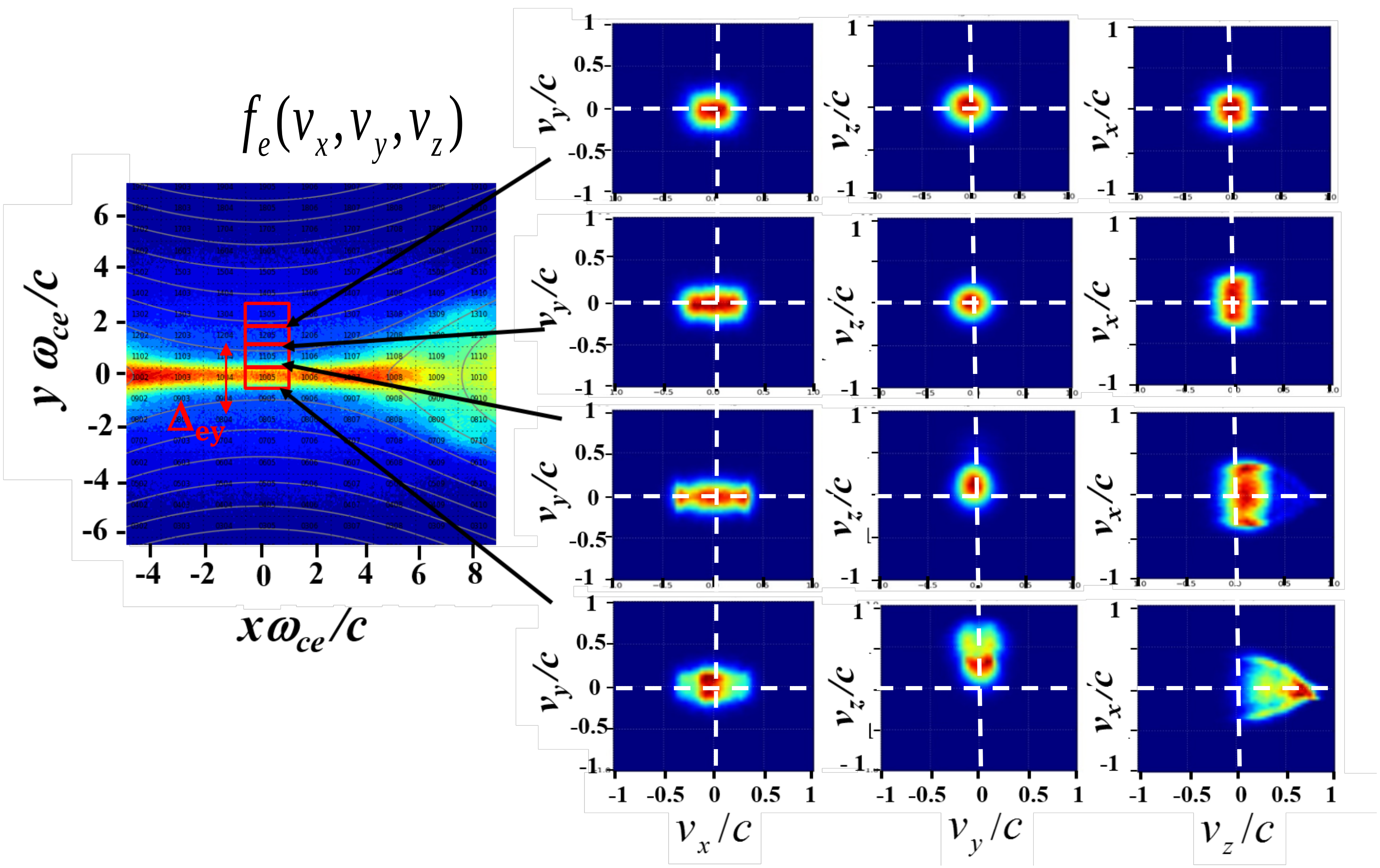


$$\hat{e}_B = \vec{B}/B \;;\; \hat{e}_2 = \vec{B}_p \times \hat{e}_z / B_p$$

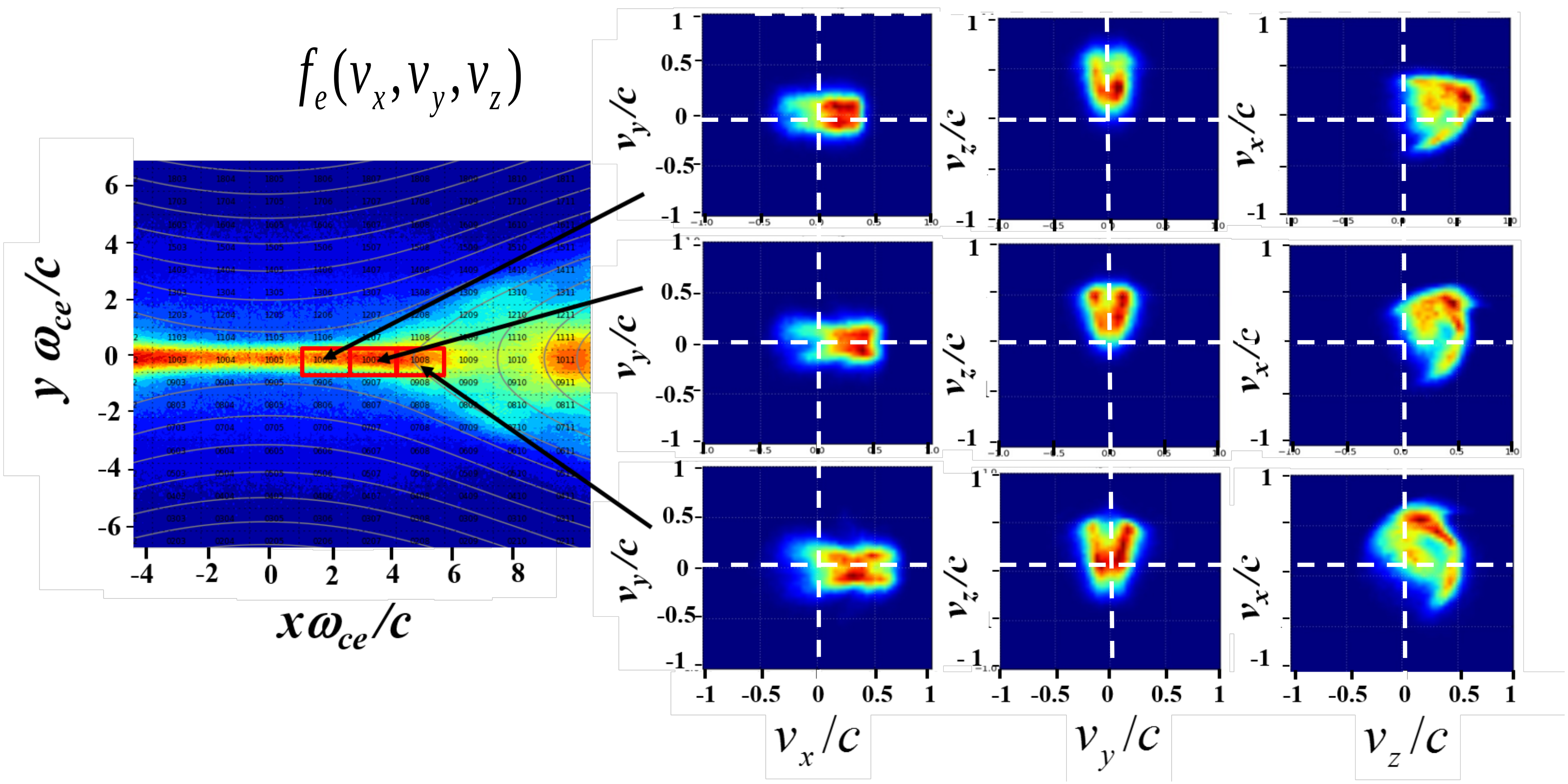
$$\hat{e}_1 = \hat{e}_2 \times \hat{e}_B \;;\; v_1 = \vec{v} \bullet \hat{e}_1 \;;$$

$$v_2 = \vec{v} \bullet \hat{e}_2 \;;\; v_\parallel = \vec{v} \bullet \hat{e}_B$$

Electron Flat-top $v_{\parallel}$-Distribution ($B_g = 0$)



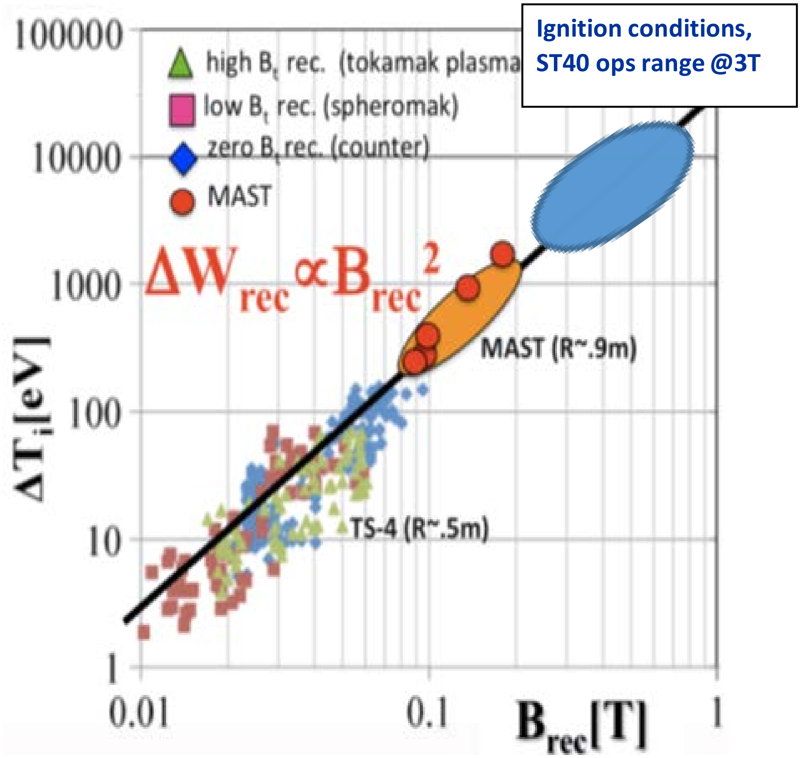
Electron Velocity Distribution in outflow region (mid-plane) (B_g=0)



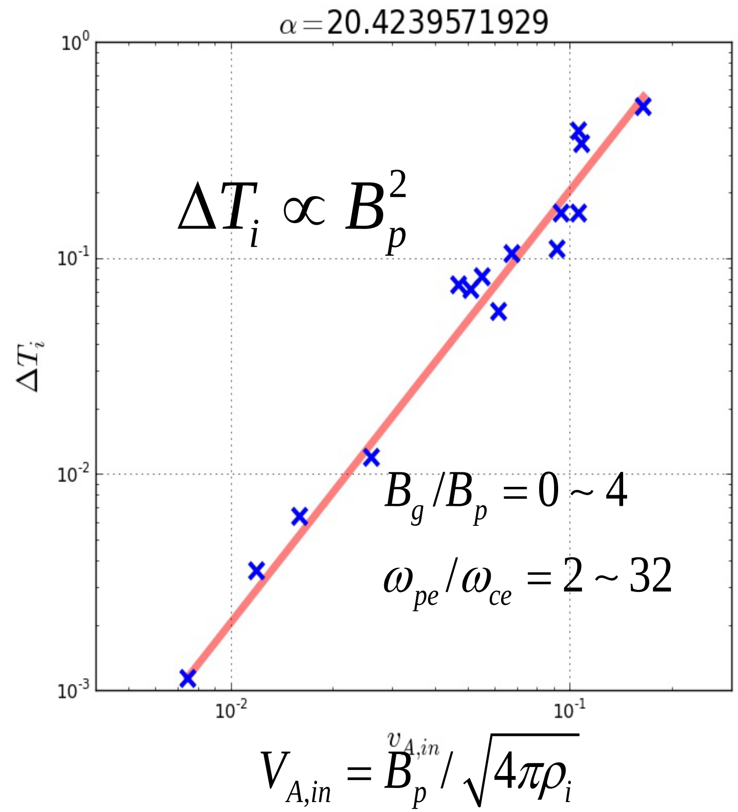
Electrons have super-Alfvenic outflow velocity and are thermalized in downstream by stronger magnetic field.

Ion Heating by Magnetic Reconnection

Experiments
Y. Ono et al.



Simulations
S. Inoue et al.



Theory has been worked out to show $\Delta T_i \propto B_p^2$

Summary on Magnetic Reconnection & flare-CME Phenomena

- Impulsively fast magnetic reconnection is induced by acceleration of plasmoid/flux rope ejection in both solar flare-CMEs and laboratory plasma merging experiments.
- Peak reconnection electric field $E_{rec} \sim O(1)$ kV/m for X-class flares.
- Electrons are accelerated in separatrix regions and inside current sheet, and they stream down along reconnected field lines to photosphere to produce hard X-ray emission.
- PIC simulations of driven magnetic reconnection show drastically different electron and ion heating/acceleration mechanisms from MHD or 2-fluid models.